1401. Take care to generate all of the extra roots at the moment you apply the arccos function.

1402. Differentiate to find the gradient of the tangent. Then find the gradient of the normal, which is the negative reciprocal. Use $y - y_1 = m(x - x_1)$ for the equation of the normal. Solve simultaneously for intersections.

1403. All three are technically possible, including (b), although three of the points are collinear, so the quadrilateral looks like a triangle.

1404. Use the chain rule, with inside function $\theta \mapsto 2\theta$, and the standard derivative for tan.

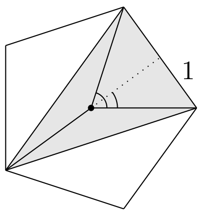
1405. Convert the percentage changes into scale factors. Take a weighted average of these scale factors, weighted by $\frac{1}{4}$ and $\frac{3}{4}$.

1406. Show that the graphs $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ are tangent at $x = a$. Then, consider the multiplicity of the root at $x = a$.

1407. This is a quadratic. Since there are square roots involved, consider carefully the validity of the roots you find.

1408. Begin with $(n-1)^2 + n^2 + (n+1)^2$.

1409. Use the following diagram:




Find the marked angles, then use right-angled trigonometry to work out the perpendicular height of the shaded triangle.

1410. You can proceed algebraically, by expanding and simplifying both side. But it is much quicker to consider the axis of symmetry of the parabola $y = \mathrm{f}(x)$.

1411. A fixed point of $x_{n+1} = \mathrm{g}(x_n)$ is a root of $x = \mathrm{g}(x)$.

1412. Look for a counterexample, considering $(\pm)^2 = +$. Remember that $\mathrm{f}^2(x)$ means “apply f twice to $x$”.

1413. Consider division by zero.

1414. Draw a $6 \times 6$ possibility space.

1415. Multiplying by a constant scales the range as you would expect. Squaring, as in part (b), renders all outputs positive.

1416. (a) Consider that the chains must lie either side of the centre of mass.
(b) Take moments around the centre of mass.

1417. Use the fact that $\log_a b$ and $\log_b a$ are reciprocal.

1418. One hour is $\frac{\pi}{6}$ radians on a clock.

1419. Show that the straight line $2x + 3y = 4$ does not pass through the unit circle.

1420. Multiply out

$$\left(x^{\frac{1}{3}} + 1\right)\left(x^{\frac{2}{3}} + px^{\frac{1}{3}} + q\right).$$

Find $p$ and $q$ such that no terms in $x^{\frac{2}{3}}$ or $x^{\frac{1}{3}}$ appear in the product.

1421. This is a quadratic in $a$.

1422. These two statements are the negations of the two usual statements in the factor theorem.

1423. The lower bound on the second largest angle is the mean of the four. For the upper bound, consider the fact that the smallest angle cannot be less than zero.

1424. (a) Evaluate the first derivative at $x = p$.
(b) Substitute $\left(p, p^3\right)$ back into $y = mx + c$, with $m$ as found in part (a).

1425. These two statements are not the same, but they have the same truth-value in this case.

1426. Consider whether or not the sum $\sum x^2$ appears in the relevant formulae.

1427. You can ignore the quadratic factor, as it is always positive. Solve $(x-1)(3x+2) \geq 0$.

1428. Sketch a Venn diagram, and identify the relevant regions. Each can be simplified to the intersection $\cap$ or union $\cup$ of two of the sets $A, B, A', B'$.

1429. Use the fact that $\log_x y$ and $\log_y x$ are reciprocals. Substitute for the latter and multiply by $\log_x y$. You’ll get a quadratic in $\log_x y$.

1430. Note that the modes of a “bimodal” population can be *local* modes, they do not have to both be *global* modes. In other words, they don’t have to have precisely the same frequency.

1431. The region is a semicircle.

1432. Sketch $y = 4 - (x-5)^2$ and $y = x$, using $\Delta$ to show that the graphs do not intersect.

1433. Solving simultaneously gives a linear equation.

1434. Use the cosine rule to find an angle, then the sine area formula. Alternatively, use Heron's formula, if you happen to know it.

1435. Set up an equation to find intersections, and then show, by factorising, that it has a double root at $x = a$.

——— Alternative Method ———

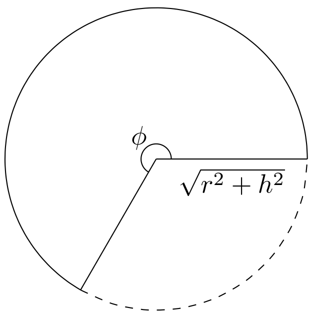
Generate the tangent directly, using calculus and the standard equation $y - y_1 = m(x - x_1)$.

1436. (a) Draw a force diagram for the load at $P$.

(b) Such questions can be answered by thinking about what would happen if the relevant beam suddenly disappeared.

1437. Consider the *throughput*. The throughput is a way of describing the output of the inside function, which is the input of the outside function.

1438. Use the following diagram:




1439. Give the integral of $y$ a name, say $\mathrm{F}(x)$.

1440. (a) Draw the half-parabola in the first quadrant, then rotate it by $90, 180, 270°$ around $O$.

(b) To rotate algebraically $180°$ around the origin, replace $(x, y)$ with $(-x, -y)$.

1441. Where there is an inlaid fraction (a fraction within a fraction), it is generally best to multiply top and bottom of the big fraction by the denominator of the little fraction.

1442. Factorise each of the equations, and consider the nature of the root of $\mathrm{f}(x) = 0$ at $x = 0$.

1443. Multiply the integrand out, noting the cancellation of $x$'s, and integrate term by term.

1444. Consider a triangle of forces.

1445. Use the factor theorem, i.e. set the expression equal to zero and solve. Then reverse engineer the factors using the roots you find.

1446. There are now two outcomes which will result in a call of six: a five which is misread upwards or a six which is read correctly. Add the relevant branch probabilities, conditioning on the true score on the die. Drawing a tree diagram might help.

1447. Start by considering the set of integers $\mathbb{Z}$. Then eliminate any values in the set that satisfy any one of the given statements.

1448. In each case, you can distribute the sum over the two terms. In other words, you can split the sum up. You get $\sum 1$, in (a), and $\sum i$, in (b). These are both standard results (they can also be derived by consideration of arithmetic progressions).

1449. The particles collide if $x_1 = x_2$. So, set up an equation and solve it.

1450. (a) Set up an equation and raise both sides to the power $b$.

(b) Show that the two sides can only be equal if $a = b = 0$, which is a contradiction.

1451. You don't need to do any calculations here. Use a symmetry argument.

1452. If $z$ is stationary with respect to $x$, then $\frac{d}{dx}(z) = 0$. Factorise the resulting equation.

1453. This is a simpler result than it looks, and is mainly about interpreting the language. It's the result called "linear coding". Integrate $\mathrm{g}'(x) = a$ to get a general linear function $\mathrm{g}(x) = ax + b$. Then, using the definition

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

you can do the algebra explicitly.

1454. Put the fractions over a common denominator:

$$\frac{36}{323} = \frac{a+b}{ab}.$$

Since $a, b$ are prime, both $\frac{36}{323}$ and $\frac{a+b}{ab}$ are in their lowest terms. So, you can equate the numerators and equate the denominators.

1455. (a) i. Using a calculator, find the probability that one weighs more than 245 grams. Square this probability.
ii. Using a calculator, find the probability that one weighs less than 250 grams. Square.
(b) Consider the applicability of the normal model and the (in)dependence of the trials.

1456. You don't need to use a compound angle formula here, although you could. Instead, divide both sides by cos, and simplify the LHS to tan.

1457. It is a true that a frictional force is created, but the student is nevertheless wrong.

1458. (a) Differentiate twice.
(b) The second derivative has a root at $x = 0$. Consider the multiplicity of this root.
(c) Over an interval $I$, a curve is concave if the second derivative is non-positive everywhere in $I$. It is convex if the second derivative is non-negative everywhere in $I$.

1459. If a fraction is zero, then its numerator is zero.

1460. Substitute a generic AP term $a_n = a + (n - 1)d$ into the formula for $b_n$.

1461. Calculate the probabilities explicitly, considering the number of successful outcomes in each case.

1462. (a) Use Pythagoras,
(b) Solve for $\mathbf{i}$ and $\mathbf{j}$ simultaneously.

1463. Perform the integrals, gathering the $+c$'s together into a single constant.

1464. Scale each exam to be out of 50, then add.

1465. Sketch $y = \tan x$ first, then consider the effect of the modulus function on those sections of the curve which are below the $x$ axis.

1466. These can be considered as a pair of simultaneous equations in $z$ and $x - y$. Use the substitution $a = x - y$, and then solve.

1467. Multiply out, find $\Delta$ and simplify.

1468. (a) Find $a$ first, using the upper two branches, then find $b$ using the third.
(b) Use $\mathbb{P}(X \cup Y') = 1 - \mathbb{P}(X' \cap Y)$.

1469. Rearrange to $\text{f}(x) = 0$ and find one (rational) root with N-R. Then, by the factor theorem, take out the relevant factor in the form $(ax + b)$. Solve the remaining quadratic exactly.

1470. The standard integral of $e^x$ is $e^x + c$. Then, for the second term, use the reverse chain rule.

1471. If two quadratics are tangent to each other, the equation formed to find their intersections must have a double root at the point of tangency.

1472. This is easier than it might look. Since $[0, 1]$ is a continuous set, the probability of equality is zero. So, there are only two directions the inequality can go, which are symmetrical.

1473. The statement isn't true; find a counterexample.

1474. Set up a triangle of forces, putting an acute angle $\theta$ between the 10 and 12 N sides. Use the cosine rule to find $\theta$, then convert to an obtuse angle.

1475. Use equations 1 and 3 to eliminate both $x$ and $z$.

1476. Split the sum up, and replace $\bar{x}$ with its definition in terms of $\sum x_i$ and $n$.

1477. The discriminants tell us that $\text{f}(x) = 0$ has exactly one root, and $\text{g}(x) = 0$ has exactly two. These may possibly be the same, however. In (b) and (c), work out whether the indices have any effect on the numbers of roots.

1478. It doesn't make any difference which face is picked first, as the octahedron is symmetrical. So, pick an arbitrary face without loss of generality, and then consider the second choice.

1479. Use the binomial expansion.

1480. (a) Use either elimination or substitution.
(b) Without this condition, the formulae in part (a) break down. Rearrange to $ad - bc \neq 0$ to see precisely how.

1481. Start with an expression: the denominator of the LHS multiplied by the RHS. Simplify to get 1.

1482. In each case, consider reflection as a replacement of the variables. In (a), replace $y$ by $-y$.

1483. Factorise immediately, without multiplying out.

1484. Set up a six by six possibility space, then restrict it according to the information given.

1485. Use index laws; it's easier if you don't convert to logarithms.

1486. Use the polynomial solver on your calculator to find the roots of the equation, and then, using the factor theorem, reverse engineer a factorisation.

1487. (a) The cables form a right-angled triangle.
   (b) You may a force diagram helpful first.

1488. Set $\arcsin x = y$ and rearrange to $x = \sin y$. Then start with the RHS of the identity.

1489. Differentiate (implicitly) term by term.

1490. This is the factor theorem in disguise.

1491. In each case, there will be either a guaranteed increase or decrease.

1492. The set of points equidistant from two points is the perpendicular bisector.

1493. Multiply up by all three denominators. You don't need to solve; you can use the discriminant.

1494. Simplify the summand, in order to ascertain what kind of series it is.

1495. Find $b$ first, then $a$, then $c$ and $d$.

1496. This is an input transformation, so ask "what has $x$ been replaced by?"

1497. Each can be considered as an arithmetic series or, alternatively, expressed in terms of the standard result $\sum i = \frac{1}{2}n(n+1)$.

1498. (a) $y = m_1 x$ intersects the rectangle at $(4, 4m_1)$. Use this point to set up an area equation.
   (b) $y = m_2 x$ intersects the rectangle at $(3/m_2, 3)$. Continue as in part (a).

1499. For a conditioning approach, multiply together six probabilities (or rather five, since the first is 1).

— Alternative Method —

For a combinatorial approach, analyse a possibility space of $6^6$ equally likely outcomes.

1500. Find the interior angle of a regular heptagon, and show that it does not divide exactly into 360°.

— End of 15th Hundred —